

UNPUBLISHED PRELIMINARY DATA

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16p. DC TO AC CONVERSION USING MAGNETORESISTANCE

OTSi*

by

and

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Magnetoresistance devices offer some attraction for use in low voltage, high current, DC to AC conversion. It is not difficult to construct devices which have an off-field resistance of less than 0.001 ohm. This off-field resistance is independent of voltage. Thus, a thousand amperes would give a 1 volt drop across the device. The major problem is in obtaining sufficiently large on-field resistance to achieve good efficiencies without major power expenditures in obtaining large magnetic fields.

1. Classical Magnetoresistance

If a current carrying conductor (or semiconductor) is placed in a magnetic field \vec{H} , the Lorentz force on a charge carrier with charge q and effective mass m is

$$\vec{F} = \frac{d\vec{p}}{dt} = q\vec{E} + \frac{q}{c} (\vec{v} \times \vec{H}) \quad (1)$$

where

 $\vec{p} = m\vec{v}$ = charge carrier momentum \vec{v} = charge carrier velocity \vec{E} = electric field c = velocity of light

For a material where the momentum of the charge carrier is governed by a finite mean free time between collisions τ then equation (1) becomes

$$\frac{d\vec{p}}{dt} = \frac{\vec{p}}{\tau} = q\vec{E} + \frac{q}{c} (\vec{v} \times \vec{H}) \quad (2)$$

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Then the current density is

$$\bar{J} = \bar{J}_n + \bar{J}_p = \sigma \bar{E} + \frac{1}{c} (\mu_p \bar{J}_p - \mu_n \bar{J}_n) \times \bar{H} \quad (3)$$

where

\bar{J}_n = electron current density

\bar{J}_p = hole current density

μ_p = hole mobility

μ_n = electron mobility

σ = conductivity

For normal magnetoresistance it has been shown that the variation in resistance resulting from a magnetic field obeys Kohler's rule.¹ Thus,

$$\frac{\rho(H) - \rho_0}{\rho_0} = F \left(\frac{H}{\rho_0} \right) \quad (4)$$

where

ρ_0 = the zero field resistivity,

$\rho(H)$ = resistivity with an applied magnetic field, H, and

F = a function depending on the physical properties of the material and the geometrical configuration of the conductor.

Making use of equations (3) and (4), it is readily shown that

$$\rho(H) = \frac{1}{\sigma} \cdot \frac{1}{1 + \frac{\mu^2 H^2}{c^2}} \quad (5)$$

¹Ziman, J. M., Electrons and Phonons, Oxford Press, London, 1960, p. 491.

The derivation of the magnetoresistance effect by this classical method is appropriate only for simple metals and degenerate semiconductors in which the energy surfaces are spherical and the relaxation time is independent of energy. The classical derivation always has as its result a quadratic dependence of the resistivity on the magnetic field, H . There are many intermetallic and semiconductor compounds which experimentally do not behave in this manner. In order to explain the galvanomagnetic phenomena peculiar to these materials, more elaborate methods of analysis must be employed.

2. The Magnetoresistance Effect in InSb

Rather than the classical quadratic dependence of magnetoresistance on magnetic field, the compound semiconductor InSb displays a change in resistivity proportional to the magnetic field. The explanation of this deviation from the classical results has been investigated theoretically by a number of authors for restricted ranges of temperature and magnetic field. In particular, it has been shown that in a strong magnetic field, electron motion normal to the field becomes quantized such that the density of electron energy levels and the scattering matrices become functions of the magnetic field.² These results have been applied to the investigation of magnetoresistance in InSb.³

²Adams, E. N. and Holstein, T. D., "Quantum Theory of Transverse Galvanomagnetic Phenomena," J. Phys. Chem. Solids, Vol. 10, p. 254-276, 1959.

³Sladek, R. J., "Magnetoresistance of High Purity InSb in the Quantum Limit," J. Phys. Chem. Solids, Vol. 16, 1960, p. 1-9.

Several different scattering models have been assumed by the various authors, but the net conclusion in each case is that for high purity n-type InSb, the magnetoresistance is a linear function of H for values of H above a few hundred gauss. In weak fields, it has been shown that the magnetoresistance varies from an H^2 dependence to a linear dependence as the field is increased from 0 to a few hundred gauss.⁴

In those applications where fields of 10 kilogauss or more are utilized, the magnetoresistance of InSb is essentially a linear function of H. That is, the resistivity of InSb can be expressed as

$$\rho_H = \rho_0 (1 + KH) \quad (6)$$

where

ρ_H = resistivity in magnetic field H,

ρ_0 = zero field resistivity,

K = proportionality constant.

In general, the proportionality constant K is primarily a function of the mobility and the temperature. It is independent of the geometry of the InSb sample. The net magnetoresistance of an InSb sample is a function of its geometry.

⁴Bate, R. T., Willardson, R. K., and Beer, A. C., "Transverse Magnetoresistance and Hall Effect in N-type InSb," J. Phys. Chem. Solids, Vol. 9, 1959, pp 119-128.

3. Configurations for Magnetoresistance Elements

The Corbino disk is one of the most common configurations used in magnetoresistance work. The zero-field resistance is given by the equation

$$R_0 = \frac{\rho_0}{2\pi t} \ln \frac{r_2}{r_1} \quad (7)$$

ρ_0 = zero field resistivity

t = thickness

r_2 = outside radius

r_1 = inside radius

With a magnetic field applied parallel to the axis of the disk, the current density will then have a component in the θ direction in addition to its radial component. Because of the geometrical symmetry, no Hall field can exist in the θ direction due to the radial component of current density. The only Hall field which can exist within the disk is parallel to its axis. This Hall field in the z direction results from the θ component of current density. Since the θ component of current density is proportional to the radial component, and the Hall field in the z direction is proportional to the θ component, the Hall field is proportional to the input current as in the case of a rectangular slab. In this situation, however, the proportionality constant relating the Hall field and the input current is small in comparison with that found in a rectangular slab. The resistance of the disk can be expressed as:

$$R = R_0 (1 + K_1 H) - K_2 F(H) \quad (8)$$

where

R_0 = zero field resistance

K_1 = proportionality constant of bulk InSb

K_2 = proportionality constant due to geometry

and $F(H)$ is a function of the magnetic field. The exact form $F(H)$ takes is questionable. The theory of the disk presented above would indicate that $F(H)$ is of the form:

$$F(H) = H + \lambda_1 H^2 \quad (9)$$

The experimental data for various disks indicate no variation of resistance proportional to H^2 for fields above a few hundred gauss. Thus, it would appear that for fields up to 22 kilogauss, the coefficient λ_1 is so small that the H^2 term can be neglected.⁵

For a rectangular slab of InSb, equation (8) can be used with different constants to account for geometry change. It should be emphasized that a minimum Hall field across the device appears to reduce the value of K_2 in equation (8). For example, placing large Hall type contacts on a rectangular slab, and shorting these contacts increases the magnetoresistance.

⁵This conclusion is valid only for high-purity n-type polycrystalline InSb. For example: Green, Milton, "Corbino Disk Magnetoresistivity Measurements on InSb," Journal of Applied Physics, Vol. 32, No. 7, 1961, lists results for single crystal InSb in which the magnetoresistance varies with H^2 for fields up to 4-6 kilogauss, varies as H for fields of 6-15 kilogauss, and then begins to approach a zero slope for fields in the vicinity of 20 kilogauss.

We have experimentally investigated the rectangular slab, the Corbino disk, and various other geometrical configurations. In general, these various specimens have been the scraps obtained from cutting rectangular slabs and Corbino disks from InSb ingots. Surprisingly enough, some of these rather odd-shaped pieces have exhibited magnetoresistance characteristics on a par with the better modified rectangular slabs and Corbino disks. These unexpected high magnetoresistances are attributed to the fact that in each case where this occurred, the geometry and contact arrangement were such that the Hall effect was minimized.

It has been shown that the classical theory of magnetoresistance as derived from the free electron model and the phenomenological equation is not applicable to high purity InSb. The failure of the classical theory is attributed to the fact that in InSb, both the density of energy levels and the scattering mechanism are functions of the magnetic field. The magnetoresistance effects predicted by quantum theory vary from a quadratic dependence upon H for low fields to a linear dependence upon H for high fields. For InSb specimens in which the transverse magnetic field is varied to a value of several kilogauss, the magnetoresistance is essentially a linear function of H .

4. Circuit Analysis

The circuit diagram of the Bridge MR converter is shown in Figure 1. This converter consists of the thermionic source, E , with internal resistance, r_s , four magnetoresistors which comprise the legs of the bridge, two magnetic field sources for stimulation of the magnetoresistors, a 1:N step-up power transformer, and the equivalent load resistance of the converter R_L .

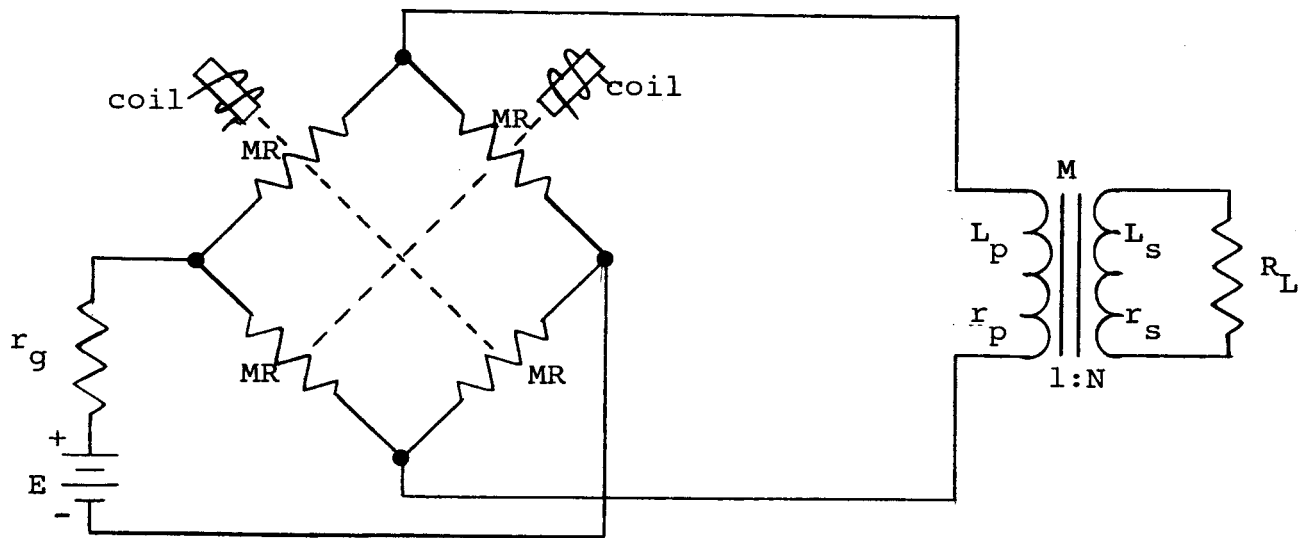


Figure 1

In this converter, the DC to AC conversion is accomplished by alternately stimulating the two pair of magnetoresistors comprising opposite legs of the bridge. This switched voltage divider action produces an alternating voltage at the output terminal pair of the bridge proper.

For square wave magnetic field stimulation, one possible electrical state of the MR Bridge is shown in Figure 2. Here the off-field resistance of each magnetoresistor is R , while it has an on-field resistance, KR .

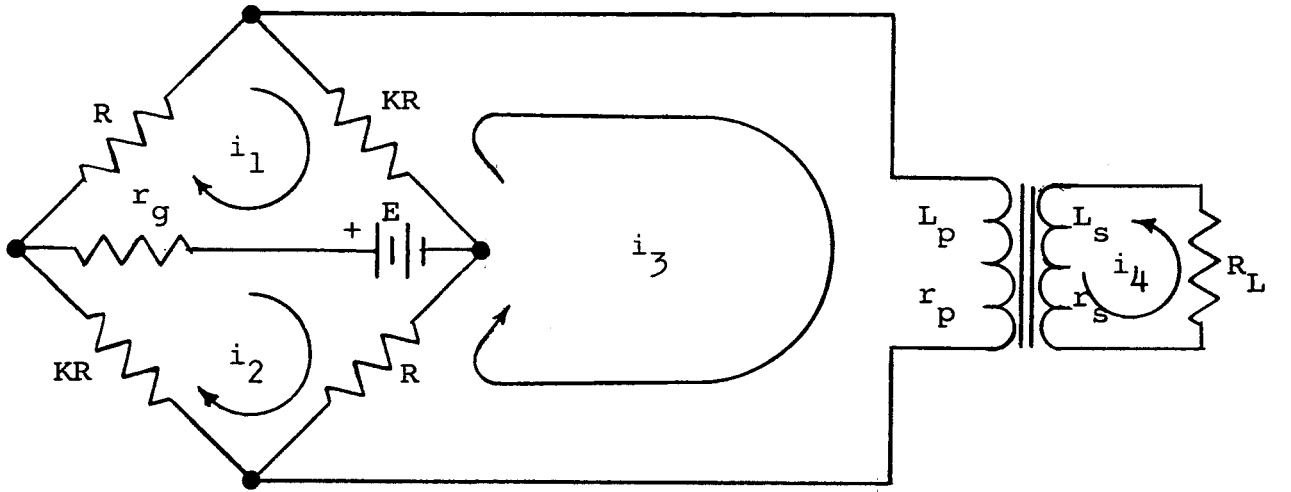


Figure 2

The mesh equations for this configuration reduce to:

$$V_e = (R_e + r_p) i_3 + L_p \frac{di_3}{dt} + M \frac{di_4}{dt} ,$$

$$0 = M \frac{di_3}{dt} + (R_L + r_s) i_4 + L_s \frac{di_4}{dt} \quad (10)$$

where:

$$R_e = R \left[\frac{2KR + r_g(K+1)}{R(K+1) + 2r_g} \right]$$

and

$$V_e = E \left[\frac{R(K-1)}{R(K+1) + 2r_g} \right]$$

An equivalent circuit, which is represented by the equation (10) is shown in Figure 3.

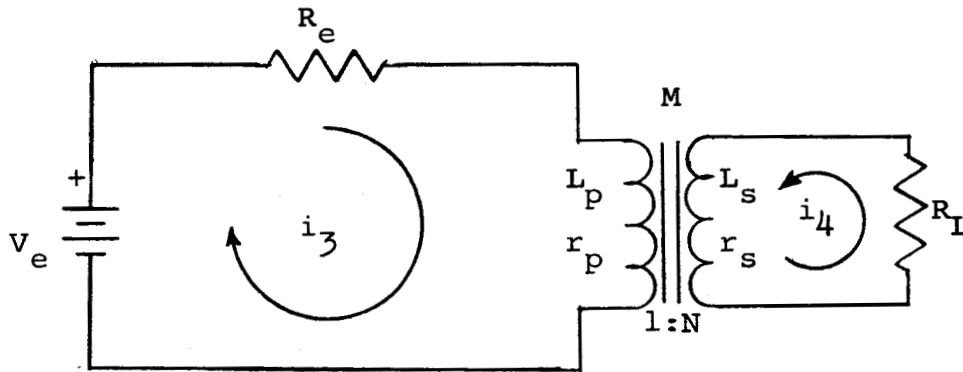


Figure 3

It should be pointed out here that the equivalent circuit of Figure 3 is valid only for calculating the currents i_3 and i_4 . The actual power input to the converter from the thermionic source, E , cannot be derived from this simplified equivalent circuit. In order to calculate the input power, it is necessary to refer to Figure 2. The current supplied by the source E is:

$$i_E = i_1 - i_2 = \frac{2E}{R(K+1) + 2r_g} + \frac{R(K-1)}{R(K+1) + 2r_g} i_3 \quad (11)$$

With this expression for source current, the input power to the converter during the first half period can be calculated.

During the second half period of magnetic field excitation the electrical roles of the two pair of magnetoresistors are reversed. The electrical state of the MR Bridge is then as shown in Figure 4.

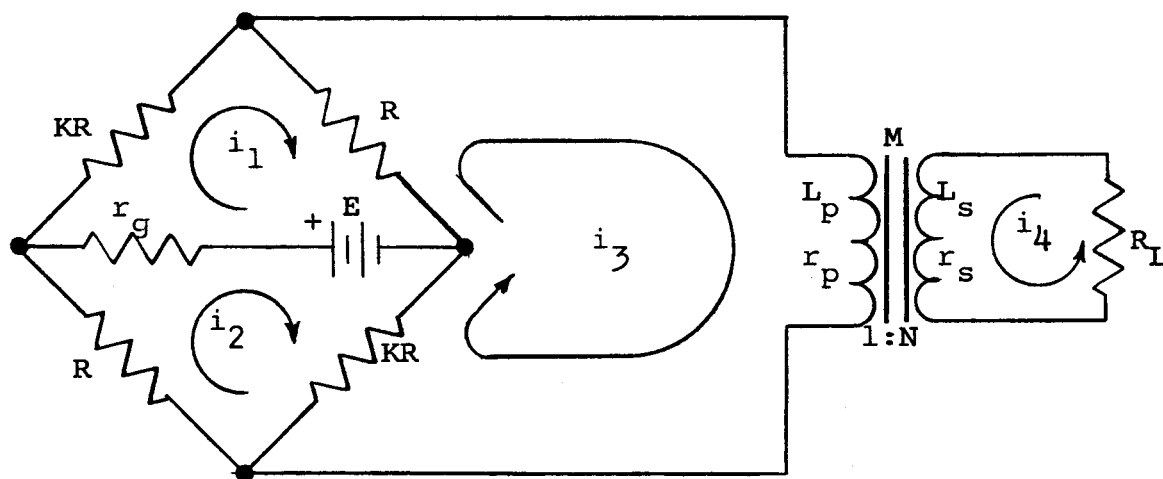


Figure 4

The simplified equivalent circuit for this case is shown in Figure 5. This circuit is identical to that of Figure 3 with the exception that the equivalent voltage source is reversed.

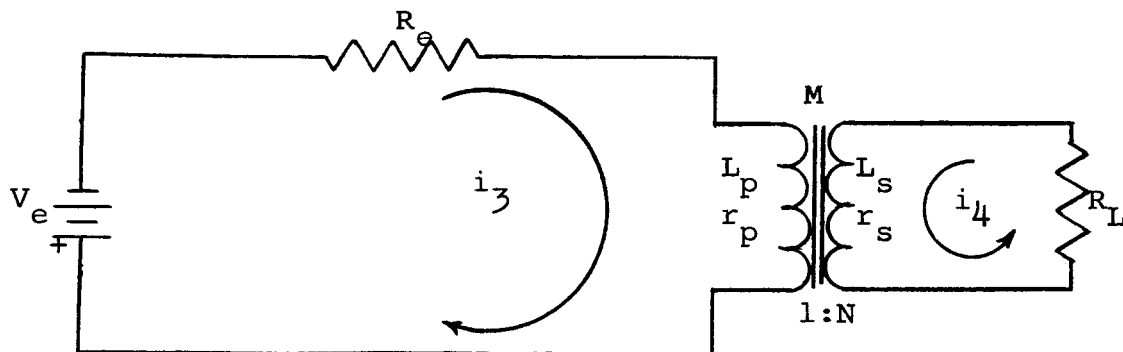


Figure 5

The equations for this circuit are:

$$\begin{aligned} -v_e &= (R_e + r_p)i_3 + L_p \frac{di_3}{dt} + M \frac{di_4}{dt} \\ 0 &= M \frac{di_3}{dt} + (R_L + r_s)i_4 + L_s \frac{di_4}{dt} \end{aligned} \quad (12)$$

During this half period of excitation, the current supplied by the thermionic source E is:

$$i_E = i_1 - i_2 = \frac{2E}{R(K+1) + 2r_g} - \frac{R(K-1)}{R(K+1) + 2r_g} i_3 \quad (13)$$

It is apparent from the forms of equations (10) and (13) that the two separate cases studied thus far may be combined into one entity. The equivalent circuit which results from this combination is shown in Figure 6.

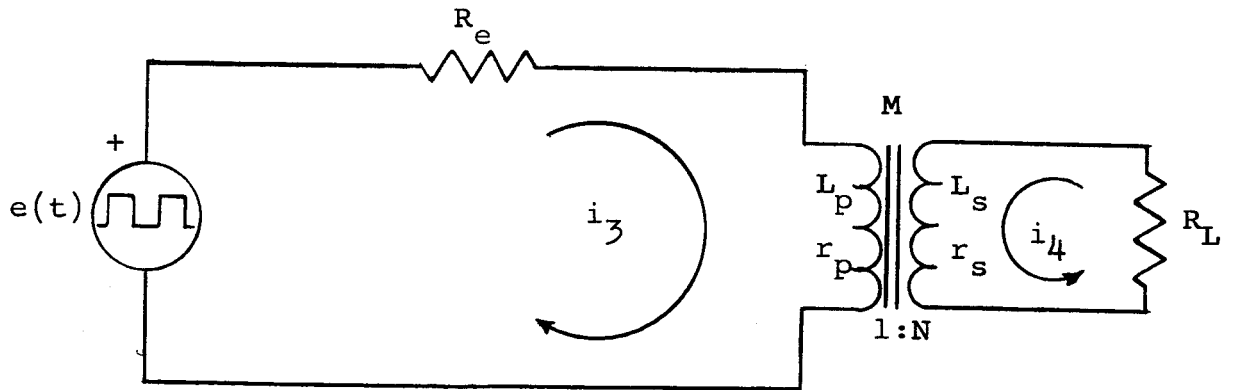


Figure 6

The set of equations which describe the electrical behavior of this circuit becomes:

$$\begin{aligned} e(t) &= (R_e + r_p)i_3 + L_p \frac{di_3}{dt} + M \frac{di_4}{dt} \\ 0 &= M \frac{di_3}{dt} + (R_L + r_s)i_4 + L_s \frac{di_4}{dt} \end{aligned} \quad (14)$$

The voltage source, $e(t)$, has the form shown in Figure 7. The period, $2a$, of the square wave is the period of the magnetic field excitation applied to the magnetoresistors.

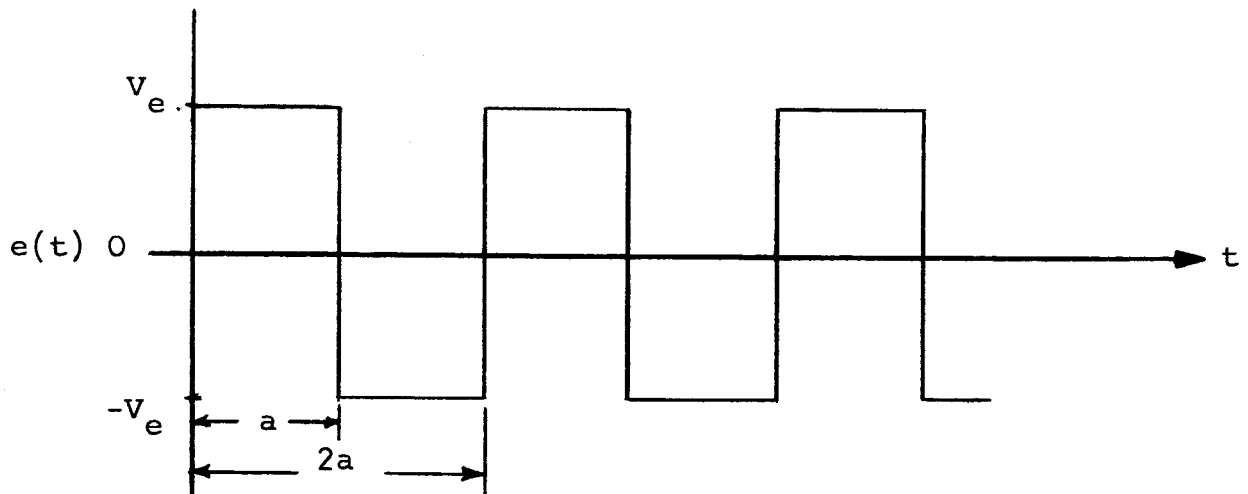


Figure 7

From the solutions for i_3 and i_4 in the pair of simultaneous differential equations (14), the input power from the source, and the power delivered to the load can be calculated.

The solutions to these equations are somewhat cumbersome for general analyses or numerical calculations by manual methods; however, the expressions are well adapted to digital computer solution. The analog computer can reduce much of the analytical computation in the evaluation process.

5. Conversion Efficiency for the Bridge MR Converter

The maximum power which the thermionic source can deliver to a load is:

$$P_{aE} = \frac{E^2}{4r_g}$$

With the inclusion of the source in the Bridge MR Converter, the maximum power which the converter can deliver to a load is:

$$P_{ac} = \frac{V_e^2}{4R_e} = \frac{E^2 R(K-1)^2}{4 [R(K+1) + 2r_g] [2KR + r_g(K+1)]} .$$

From this expression, it is apparent that the maximum power which the converter can supply to a load is less than that available from the source. This detrimental aspect of the converter circuit is minimized for the magnetoresistor having the properties:

$$K \gg 1$$

$$2R \ll r_g$$

$$KR \gg 2r_g$$

Under these conditions, the conversion efficiency approaches the value:

$$\zeta = \frac{P_{ac}}{P_{aE}} = 100\%$$

A plot of efficiency versus magnetoresistance ratio is shown in Figure 8. This was obtained by analog computer solution of the MR Bridge equations with appropriate scaling.

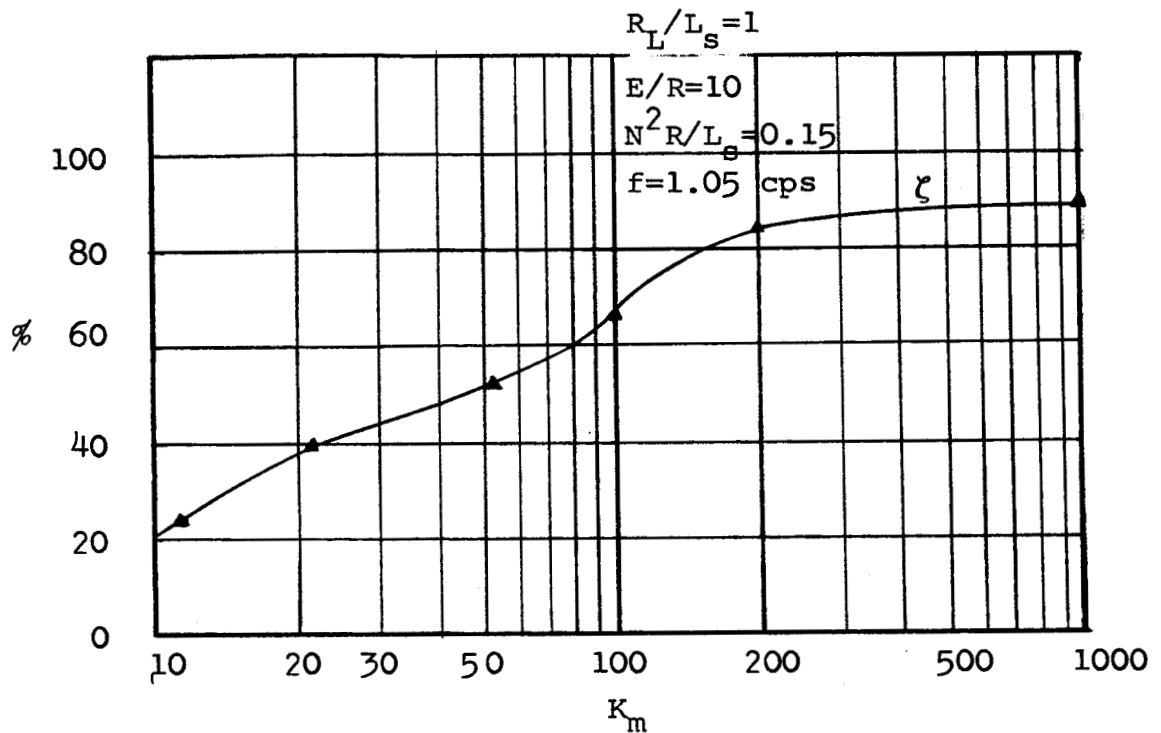


Figure 8

6. Magnetoresistance Experiments

A number of different geometrical shapes, for both bismuth and indium antimonide, have been tested for their magnetoresistance properties. None were found to be consistently better than the Corbino disk, although some came quite close. If converter design warrants, the Corbino disk could be replaced by a more adaptable geometry if a small loss in the magnetoresistance ratio could be tolerated. Some geometries were found for which this loss would amount to only two or three percent.

Some of the more interesting geometrical experiments are represented by the specimens illustrated in Figure 9.

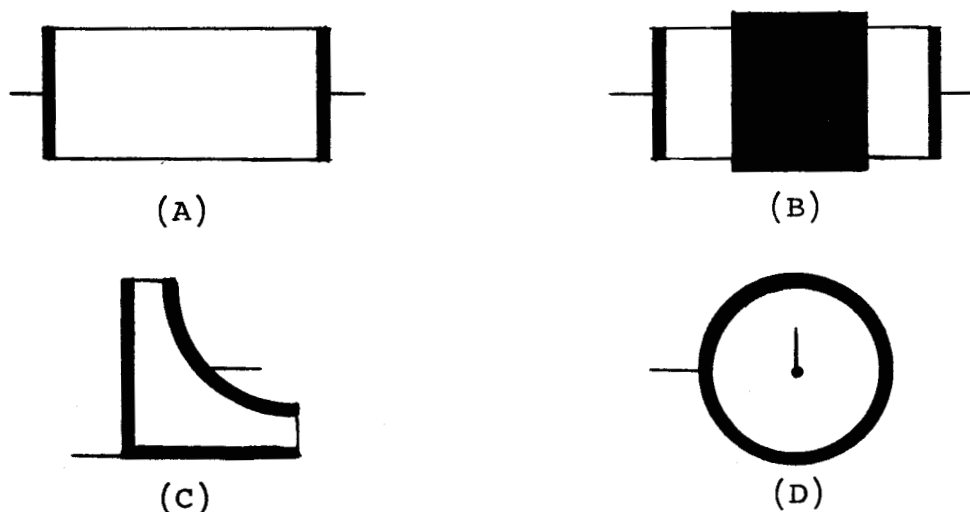


Figure 9

The results listed in Table 1 are typical for these geometries.

Table 1

Representative Test Data for the Geometries Shown in Figure 9

Sample	I	T($^{\circ}$ K)	H(kg)	R/R_0	R_0 (ohms)
A(InSb)	1 amp	300	21.5	6.15	0.035
B(InSb)	1 amp	300	21.5	22.5	0.016
C(InSb)	1 amp	300	21.5	50.0	0.0035
D(InSb)	1 amp	300	20.0	51.12	0.0041
D(Bi)	10 amp	80	20.0	145.0	0.012

Note that the MR ratio (R/R_0) for specimen C is nearly that found for the Corbino disk (specimen D). This unexpected high magnetoresistance for specimen C is attributed to the fact that the geometry and contact arrangement is such that the Hall effect is minimized.